Joint Probability Densities and Quadrant Contributions in a Heated Turbulent Round Jet

K. R. Sreenivasan* and R. A. Antonia†
University of Newcastle, Newcastle, N.S.W., Australia

Abstract

CONDITIONAL (turbulent) isodensity contours and cross sections of the joint probability density functions of the axial (u) and radial (v) velocity fluctuations, and temperature fluctuation (θ) are obtained in a slightly heated round jet with a coflowing external stream. Also obtained are the occupancy times in various quadrants and the quadrant contributions to average fluxes $\langle uv \rangle$, $\langle u\theta \rangle$, and $\langle v\theta \rangle$. Comparison with corresponding quantities of the appropriate bivariate Gaussian density functions shows that departures from Gaussianity are less pronounced for the outward radial motion than for the inward radial motion associated with the possibly newly entrained fluid.

Contents

Previous measurements[1,2] in axisymmetric jets have shown that turbulent fluctuations are closely Gaussian near the axis but become increasingly non-Gaussian in the intermittent region. Although fluctuations in the turbulent region only of the flow (designated below as conditional quantities) are known to be more closely Gaussian than the conventional fluctuations, significant departures from Gaussian flow persist.[3] The objective of this paper is to explore these departures in some detail in the case of a slightly heated axisymmetric jet with a coflowing external stream, and to relate these departures to the physical processes occurring in the flow.

The jet velocity at the exit of the nozzle (diameter ≃ 2 cm) was 32 ms^{-1}, and that of the coflowing stream was 4.8 ms^{-1}. All measurements were made at a streamwise station 59 diameters downstream of the nozzle exit. Velocity fluctuations were measured with an X wire, and temperature fluctuation was measured with a cold wire (1-μm diameter operated at 0.1 mA) located at about 1 mm from the midpoint of the X wire. Further details on experimental conditions and signal processing can be found in Ref. 3.

Isodensity contours of the joint density functions of pairs of fluctuations (u_t, v_t), (v_t, θ_t), and (u_t, θ_t) were obtained from measured joint density functions; here, suffix t indicates conditional quantities with respect to their own means. They were then compared with the corresponding isodensity contours of the vibariate Gaussian density function. "Conditional" averages $\tilde{u}_t$, $\tilde{v}_t$, and $\tilde{\theta}_t$ were also obtained according to relations similar to

$$\tilde{u}_t \mid_{v_t=\alpha} = \int_{-\infty}^{\infty} u_t p(u_t, \alpha) \mathrm{d}u_t \tag{1}$$

For the Gaussian density function, these "conditional" averages are simply straight lines of slopes r and r^{-1} passing through the origin, where r is the correlation coefficient for the pair of fluctuations considered.

Close to the jet axis, departures from Gaussian behavior are not very large, and isodensity contours are essentially the same as for conventional fluctuations given in Refs. 1 and 2. For this reason, they are not given here. In the intermittent region, most significant departures from Gaussian behavior occur in quadrants $(x<0, y<0)$ and $(x>0, y>0)$, where x and y denote u_t, v_t, or θ_t. This is especially clear in the quadrants involving negative θ_t, because of the sharp cutoff in temperature near the ambient value; see Figs. 1 and 2, which correspond to a radial position η_1 where $\gamma \simeq 0.28$. Contours spread out in $(x>0, y>0)$ quadrants and peak together in quadrants $(x<0, y<0)$, both of which thus contribute a larger fraction to the average product $\langle xy \rangle$ than the other two quadrants. Gaussian "conditional" averages are rather poor approximations to measured $\tilde{u}_t$, $\tilde{v}_t$, and $\tilde{\theta}_t$. Figure 3 shows cross sections of the joint density function $p(v_t, \theta_t)$ taken for five constant values of v_t (0, ±1, ±2 standard deviations) at a radial position η_2 where $\gamma \simeq 0.9$. Clearly, turbulent motion

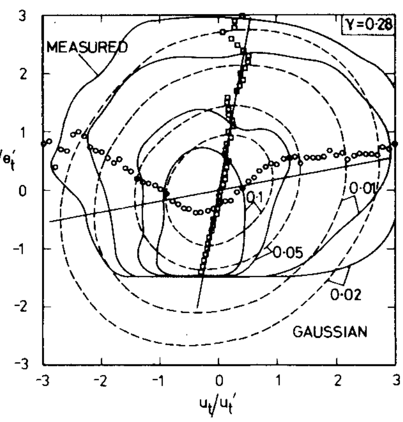

Fig. 1 Isodensity contours of $p(u_t, \theta_t)$ and "conditional" averages. □ = $\tilde{u}_t$; ○ = $\tilde{\theta}_t$. Straight lines with slopes r and r^{-1} correspond to "conditional" averages for the joint Gaussian density function, $r = 0.38$.

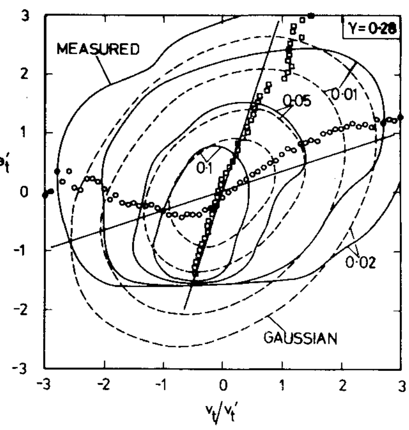

Fig. 2 Isodensity contours of $p(v_t, \theta_t)$ and "conditional" averages □ = $\tilde{v}_t$; ○ = $\tilde{\theta}_t$, $r = 0.44$.

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*Professional Officer, Dept. of Mechanical Engineering.

†Professor of Mechanical Engineering, Dept. of Mechanical Engineering.

Table 1 Occupancy times and quadrant contributions.[a]

			Percentage occupancy time in the quadrant				Percentage contribution to $\langle xy \rangle$ from the quadrant			
	x	y	$x>0$ $y>0$	$x>0$ $y<0$	$x<0$ $y<0$	$x<0$ $y>0$	$x>0$ $y>0$	$x>0$ $y<0$	$x<0$ $y<0$	$x<0$ $y>0$
η_2	u	v	30	17	37)	16	72	−18	64	−18
			(31)	(19)	(31)	(19)	(70)	(−20)	(70)	(−20)
	u	θ	32	15	34	19	70	−12	63	−21
			(32)	(18)	(32)	(18)	(65)	(−15)	(65)	(−15)
	v	θ	33	13	36	18	72	−9	54	−17
			(33)	(17)	(33)	(17)	(63)	(−13)	(63)	(−13)
	u_t	v_t	30	19	34	17	75	−23	71	−23
			(31)	(19)	(31)	(19)	(72)	(−22)	(72)	(−22)
	u_t	θ_t	31	18	32	19	76	−19	68	−25
			(31)	(19)	(31)	(19)	(70)	(−20)	(70)	(−20)
	v_t	θ_t	32	15	35	18	71	−11	56	−16
			(32)	(18)	(32)	(18)	(65)	(−15)	(65)	(−15)
η_l	u	v	20	23	38	19	118	−29	50	−39
			(29)	(21)	(29)	(21)	(84)	(−34)	(84)	(−34)
	u	θ	13	30	45	12	152	−24	40	−68
			(29)	(21)	(29)	(21)	(90)	(−40)	(90)	(−40)
	v	θ	14	25	49	12	112	−12	27	−27
			(32)	(18)	(32)	(18)	(67)	(−17)	(67)	(−17)
	u_t	v_t	23	19	39	19	108	−31	65	−42
			(29)	(21)	(29)	(21)	(91)	(−41)	(91)	(−41)
	u_t	θ_t	24	19	34	23	140	−50	90	−80
			(28)	(22)	(28)	(22)	(115)	(−65)	(115)	(−65)
	v_t	θ_t	27	15	39	19	83	−12	51	−22
			(30)	(20)	(30)	(20)	(77)	(−27)	(77)	(−27)

[a] Numbers in parentheses refer to Gaussian values.

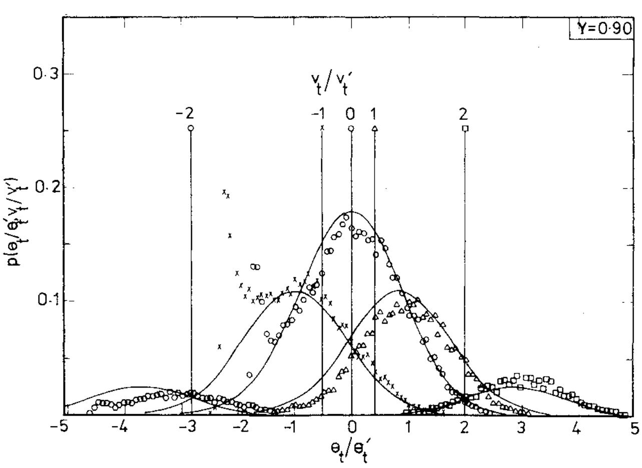

Fig. 3 Cuts of conditional joint density function $p(v_t, \theta_t)$ for different values of v_t. (Note shifted origin for the abscissa.) Curves correspond to joint Gaussian function, $r = 0.44$.

associated with negative θ_t, especially when v_t is also negative, is distinctly non-Gaussian, while that related to positive v_t and positive θ_t is essentially Gaussian. Cross sections of $p(u_t, v_t)$ show that the motion associated with $u_t>0$ is more Gaussian than that associated with $u_t<0$. That is, the radial motion of fluid originating in the freestream, especially when it is inward and associated with streamwise deceleration, is non-Gaussian, while the motion associated with accelerating outward radial motion is largely Gaussian.

Table 1 gives, for η_l and η_2, the occupancy times (i.e., the fraction of time a pair of fluctuations (x,y) can be found in a given quadrant of the xy plane) and the fractional contribution to $\langle xy \rangle$ from that quadrant. Data correspond to both conventional and conditional fluctuations. At η_2, large contributions of the order of 70% to $\langle xy \rangle$ occur in each of the quadrants $(x<0, y<0)$ and $(x>0, y>0)$, while the occupancy times in each of these quadrants is about 30% of the total time. The other two quadrants provide negative contributions of the order of 20% each to $\langle xy \rangle$. Gaussian results are good approximations to measured values, and the differences between the conventional and conditional estimates are small (because $\gamma \simeq 1$). At η_l $(\gamma \simeq 0.28)$, the contribution to $\langle xy \rangle$ from the quadrant $(x>0, y>0)$ is especially pronounced, and is generally more than 100% of $\langle xy \rangle$ (although its occupancy time is only 20%), while that of the quadrant $(x<0, y<0)$ has decreased. Consistent with our previous observations, quadrants $(x>0, y>0)$ are closely Gaussian in the conditional case, while most significant departures from Gaussianity occur in quadrants associated with the inward radial motion $(v_t<0, \theta_t<0)$. In general, conventional quantities are rather poorly approximated by the Gaussian estimates.

Finally, it is useful to identify the quadrants $(u>0, v>0)$ and $(u<0, v<0)$, respectively, with an outward "ejection" of *high* momentum fluid and a "sweep" of relatively *low* momentum fluid, although this classification is somewhat oversimplified in an axisymmetric flow (where ejection and sweep events may be associated with a small azimuthal component). The other two quadrants $(u<0, v>0)$ and $(u>0, v<0)$ can be termed "interaction" events. At η_2 (where the production of turbulent energy is nearly maximum), quadrant occupancy times and contributions from ejection and sweep events are consistent with the corresponding results in the region of maximum turbulence production in wall shear flows.[4]

Acknowledgment

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